Correlations In Magnitude and Nature of Energy Gap At The Dirac Point For Magnetic Topological Insulator and Rest Energy Of Electron

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Abstract
Search and analysis of scaled correlations and analogies in the opening of the energy gap at the Dirac point in the topological state dispersions for magnetic topological insulator with the rest energy of an electron have been carried out. It was done taking into account the replacement of the electron velocity at the Fermi level with the speed of light and the use of the relationship between the energy gap and the size of magnetic vortex excitations in magnetic topological insulator, on the one hand, and the rest energy for a free electron and the Compton wavelength, on the other hand. Based on this analogy, an assumption was made about possible complex internal dyon-like structure of electron as a local topological medium limited by the dimensions associated with the Compton wavelength and a virtual magnetic vortex-like texture (magnetic charge) connected with electron charge inside. An attempt was made to analyze what types of parameters and processes in this representation can determine the value of the rest energy (and mass) of electron, which make it possible to combine the wave properties of electron with the rest energy.

Keywords: Magnetic topological insulators, Dispersion of topological States, Energy gap at the dirac point, Electron rest energy

INTRODUCTION
A large number of recent scientific publications indicate that solid state physics can serve as a good testing platform for studying effects from other areas of physics, such as quantum electrodynamics, high energy physics, relativistic and astrophysics, due to their possible correlations with many-body effects and their quasi-particle representation, which are actively studied and used in solid state physics. One of the well-known examples of such correlations can be a graphene, which is characterized by linear dispersion of the Dirac cone states, similar to that for photon with zero effective mass [1, 2]. Replacing the speed of light with the velocity of electrons at the Fermi level allows us to find direct correlations with some effects studied and predicted in high-energy physics and quantum electrodynamics (such as the Klein paradox [3] and others). It is the lower velocity of electrons at the Fermi level (in graphene it is 1/300 of the speed of light) and the increased effective size of quasi-particles

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formed to describe many-body interactions in graphene that allows us to go to the description of such effects in graphene at much lower energies.

Another similar material is a topological insulator (TI), which is also characterized by linear dispersion of the Dirac cone topological surface states due to a change in the topology at the interface between TI and vacuum (i.e. at the TI surface), see, for example, [4-6]. The existence of the Dirac cone states is determined by the laws of topological transformations provided by the preserving of the time reversal symmetry (TRS). It is assumed that TIs can be a good testing platform for analysis of the ideas of the axion fields and interactions in high-energy physics and quantum field theory [7,8], the problem of existence of an axion as a carrier of dark energy in cosmology [9], search of a magnetic monopole [10-14] and a dyon [10-12,15-17] and possibility of realizing the Witten effect [15-18]. At the same time, last years an interest in study of magnetic TIs has been significantly increased, where the TRS condition is violated due to influence of the internal magnetic field. It is followed by opening the energy gap at the Dirac point (DP) in the topological surface state (TSS) dispersion. As a result, these materials offer the possibility of realizing a number of unique topological effects, such as the quantum anomalous Hall effect, the topological magnetoelectric effect, etc. (see, for example, [4-7]). The gap size increases with the developed internal effective magnetic field affecting the topological states [4-6,19-23] and can vary from units of meV to 30-60 meV, depending on the level of doping with a magnetic metal. In this case, magnetic impurities inside TI form a magnetic texture [22-27], the interaction with which opens the gap at the DP. At the same time, a new class of intrinsic magnetically ordered TIs was recently discovered and successfully synthesized (for instance, with the stoichiometry of MnBi$_2$Te$_4$ [28-33]), which is characterized by the higher gap value, up to 80-90 meV (according to theoretical calculations [28-33]) and up to 60-70 meV (in accordance with experimental estimates [28,32,33], related to the developed internal magnetic field.

Opening of the energy gap at the DP is described by the formation of the "mass" term in the TSS dispersion $E_k = \pm (\vartheta_F^2 \hbar^2 k^2 + \Delta_{ex}^2)^{1/2}$ (see, for instance, [4-6,27]). When the Fermi level electron velocity ($\vartheta_F$) is replaced by the speed of light ($c$), this expression correlates to some extent with the formation of the "mass" term in the Klein-Gordon-Fock (K-G-F) equation ($E^2 = c^2 p^2 + (mc^2)^2$), describing the value of the electron rest energy, determined by its mass (see, for example, [34,35]). It should be noted here that the K-G-F equation was originally written for a spin-less particles under condition of the TRS preserving. However, according to modern interpretation in the complex representation, this equation already corresponds to excitation of particles and antiparticles with quantum numbers opposite in sign and the formation of the corresponding conserved electron charge [36,37]. It is such interpretation with the TRS violation that will be used in the current article below.

In relation to possibility of such scaled correlations in the size of the DP gap, we may also assume that the scaled correlations are also possible in physical processes and interactions that determine the size of the DP gap in magnetic TIs and the electron rest energy. An analysis of such correlations can help, among other things, a more detailed understanding of the problem of the simultaneous formation of the rest energy (and mass) and the wave properties of an electron. Unfortunately, the modern theory (see, for example, [36]) does not give a direct answer to the question, what and which known parameters can determine the rest energy and mass of electron. In the Higgs model, the mass of an electron (as an elementary particle) is associated with the interaction of particles with a scalar Higgs field, without analyzing the possible internal structure. At the same time, a direct estimate of what and which parameters determines the electron rest energy value, as well as the concept of the corresponding physical laws, have not yet been developed in detail.
In this work, we will try to draw an analogy in the magnitude of the energy gap open at the DP and the doubled electron rest energy in the dispersion dependence of a free electron. At the same time, assuming also an analogy with the formation of the corresponding gap, we will show, what "finger" considerations and concepts can be considered when analyzing a formation of the energy gap in the dispersion dependence of a free electron, as a supposed local topological object with a complex dyon-like internal structure. As an object for comparison and analysis of such correlations, we have chosen the magnetic topological insulator MnBi$_2$Te$_4$, characterized by the size of the DP gap of about 55-80 meV (see below).

![Figure 1. Schematic comparison of the rest energy of electron ($m_0c^2$) in the dispersion dependence $E(p)$ - (a) and the energy gap open in the TSS at the DP for magnetic TI ($\Delta = 2m_0c^2$) - (b).](image)

Figure 1 shows a schematic comparison between the electron rest energy ($m_0c^2$) in the dispersion dependence $E(p)$, equals to $0.51 \times 2 = 1.02$ MeV, and the energy gap open at the DP in the TSS dispersion in magnetic TIs ($\Delta = 2m_0c^2$). When analyzing such a correlation, we will proceed from the fact that the size of the DP gap in a magnetic TI is determined by the interaction of two Dirac fermions, initially characterized by linear gapless dispersions, with an effective magnetic excitation with the vortex-like texture formed in a magnetic TI. A similar idea was expressed in Refs [22-24] for magnetically-doped TIs, when the magnetic interaction opens the energy gap at the DP, albeit of a smaller size. The electron rest energy (within the framework of the analogies noted above) can also be interpreted as half of the energy gap [34,35] formed between states with positive and negative energies during the interaction of the Dirac fermions with internal “magnetic disturbance or excitation”. In both cases, the size of the gap at the DP and the electron rest energy are described by the same type of hyperbolic equations of the K-G-F type (see below), only with a difference in the values of the speed of light and the speed of electrons at the Fermi level ($\theta_F$), which in MnBi$_2$Te$_4$ is of about 5.5 - 10$^5$ m/sec [31]. Accordingly, the value of $\hbar \theta_F$ in MnBi$_2$Te$_4$ is of about 2.3–2.5 eV Å [38], which is much less than the value $hc = 12398/6.28 = 1974.2$ eV Å.

**EXPERIMENTAL RESULTS**

Figs. 2.a, b show the dispersion dependences measured for magnetic TI MnBi$_2$Te$_4$ by angle-resolved photoemission spectroscopy in the region of the TSS close to the DP, where the energy gap opens.
Figure 2. (a, b) – The TSS ARPES dispersion dependences for MnBi$_2$Te$_4$ in the N(E) and d$^2$N/dE$^2$ forms, measured using synchrotron and laser radiation with energies of 6.3 and 28 eV, respectively. (c) - the corresponding EDC in the region of the DP (at $k_H = 0$), with decomposition into spectral components, which show the formation and size of the energy gap at the DP. (d) - Changes in the energy splitting between the upper and lower parts of the Dirac cone with variation of $k_H$ relative to the $\Gamma$ point. The smallest value corresponds to the size of the DP gap.

The spectra were measured using synchrotron and laser radiation with energies of 6.3 eV and 28 eV, respectively. The dependences are presented in the form of N(E) - (a) and d$^2$N/dE$^2$ - (b) for better visualization of the gap open at the DP. As we noted above, this material is characterized by an anomalously large gap, which, according to theoretical estimates [28], can reach 88 meV. Column (c) presents the decomposition into components showing the formation of the gap at the DP with the sizes between 55 and 80 meV. Column (d) shows the results of similar estimates of the splitting between the states of the upper and lower parts of the Dirac cone for different values of $k_H$ passing through the $\Gamma$-point ($k_H = 0$). The minimum splitting value determines the size of the gap, which coincides with that shown in the decomposition in column (c). The presented experimental dispersion dependences are similar to those measured in [28, 32, 33, 24] and, on the whole, demonstrate a gap size of at least 55–80 meV, which correlates with the theoretical estimates.
It should be noted that despite the fact that the size of the gap is determined by the effective magnetic moment in the region of spatial localization of the surface Dirac cone topological states, the gap remains open even above the Néel temperature (24.5 K for MnBi$_2$Te$_4$) [28,32,33]. It means that the interaction that opens the energy gap at the DP has rather short-range character.

A similar behavior of the DP gap is also observed for magnetically-doped TIs, where the gap also remains open above the Néel temperature [21,22].

To explain this behavior of the DP gap, it was suggested in [22-24] that the formation of the energy gap in magnetic TIs can be caused by magnetic fluctuations or vortices (of the skyrmion (or meron) type) and their interaction with the Dirac fermions. Magnetic perturbation connects the Dirac fermions of opposite chirality (with opposite spin orientation), which opens the gap at the DP for the initially gapless linear TSS dispersions characteristic of nonmagnetic TI.

**DISCUSSION**

Below we will try to find and use the analogy between the gap open at the DP for a magnetic TI and the dispersion relation for a free electron, which is characterized by the electron rest energy (determined by its mass ($m_o c^2$)). As was already noted, the electron rest energy can also be considered as a half of the energy gap opened by the interaction between the initially massless Dirac fermions and some magnetically-like excitation (see Figure-1). This assumption may be based on the fact that, at certain conditions, an electron (within the framework of solid state physics) is considered as a plane wave constructed from the oppositely directed running plane waves, which can form the energy gap in the dispersion dependence under magnetic field and can be characterized by corresponding effective mass.

It is known that for a free electron the magnitude of the rest energy can be obtained from an equation similar to the K-G-F equation using the expression for the Compton wavelength of an electron [27,34,35]: \( \Psi^2 \Psi - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Psi = \left( \frac{mc}{\hbar} \right)^2 \). The resulting dispersion dependence can be written as \( E^2 = c^2 p^2 + \left( \frac{mc^2}{\hbar} \right)^2 \), where \( \frac{\hbar}{mc} = \lambda_c^* \) corresponds to the reduced Compton wavelength of the electron \( \lambda_c^* = \frac{\lambda_c}{2\pi} = \frac{2.426 \times 10^{-2} \text{Å}}{2\pi} = 3.86 \times 10^{-3} \text{Å} \). In this case, the electron rest energy (and, accordingly, the electron mass) can be represented as \( m_o c^2 = \frac{\hbar c}{\lambda_c^*} = \frac{\hbar c}{\lambda_c} 2\pi = 0.511 \text{ MeV} \).

Using the schematic analogy between appearance of the "energy gap" in the dispersion dependence for a free electron (\( \Delta = 2m_o c^2 \)) (Figure-1) and opening of the gap at the DP in the dispersion dependences \( E(k) \) for magnetic TI MnBi$_2$Te$_4$ (Figure-2), we will also assume a possible correlation in the processes leading to the opening of the energy gap and transformation from a linear dispersion dependence to a hyperbolic one in the region of the open gap. As we noted earlier, a direct correlation between the electron rest energy and the size of the energy gap open in a magnetic TI can be carried out by replacing the speed of light with the speed electrons at the Fermi level (\( \theta_F \)) with simultaneous replacing the Compton wavelength \( (r_c = \frac{\lambda_c}{2}) \) with the effective radius of the formed magnetic skyrmion-like vortices. For MnBi$_2$Te$_4$, the value of \( \hbar \theta_F \) can be estimated from the dispersion dependences shown in Figure 2 at the level of 2.5–2.7 eV Å, which correlates with those (2.3–2.5 eV Å) presented in [31,38,39]. Such correlations can also be based on the results of [40, 41], where it was noted that for skyrmion-like excitations in magnetic TIs a relationship exists between the effective (critical) skyrmion radius, the Fermi level velocity, and the exchange interaction between surface electrons and magnetic excitation \( \Delta (R \sim \frac{\hbar \theta_F}{2}) \). In this case, the very formation of magnetic vortex-
like spin texture around magnetic impurity is determined by the response of a topological medium to a magnetic perturbation (see, for example, [22, 24, 25]), what is inherent in a magnetic TI. The same takes place for a Rashba electron gas with a large spin-orbit interaction [26]. Unfortunately, the effective diameter of the skyrmion (or meron) type magnetic excitations formed in magnetic TI MnBi$_2$Te$_4$ is not known exactly at the moment. Although a possibility of formation of skyrmion excitations in both MnTe and MnBi$_2$Te$_4$ was noted in [42–43] and [39, 44], respectively. At the same time, in the theoretical work [44], the periodicity formed in the network of skyrmion excitations in the system based on MnBi$_2$Te$_4$ (and, accordingly, the diameter of skyrmion excitations) was estimated at a level of 4-5 $a_0$, i.e. ~ 2-2.5 nm. Moreover, in experimental works (see, for example, [45]), the diameter of skyrmion-like excitations in various compounds, including those based on Mn, was estimated on average from 2-3 to 200 nm. In this case, if instead of the Compton wavelength we substitute in the expression for the energy gap the averaged value of the effective diameter of vortex skyrmion-like excitations, presumably formed in a magnetic TI, at a level of 2-200 nm, we can obtain the “scaled” value of the energy gap $\Delta \sim \frac{\hbar q_r}{R} = \frac{(2.3-2.7) (eV \cdot Å)}{(20-2000)Å}$ in the range ~ 100 - 1 meV. These values are within the experimentally measured values of the DP gap for various magnetic TIs [28-33, 19-24], including those shown in Figure 2.

Thus, the presented estimates make it possible to compare, at least formally, the opening of the energy gap at the DP for a magnetic TI (with a certain “scaled” coefficient) with the formation of the "energy gap" in the dispersion dependence for a free electron, the value of which is equal to the doubled electron rest energy (see Figure-1) - 0.51x2 = 1.02 GeV. This analogy (correlation) allows us to suggest that the energy gap in the dispersion dependence of a free electron can also be interpreted as caused by the interaction of initially massless Dirac fermions of opposite chirality with a magnetically-like vortex texture, presumably localized inside an electron. In this case, we can suggest that an electron itself can also be considered as a topological medium with the size limited by the Compton wavelength. Due to the localization of such magnetically-derived vortex texture inside electron (as local topological medium) this texture does not appear directly in the experiment, but determines the rest energy (and mass) of the electron, see below. At the same time, by analogy with magnetic TIs (as an axion (topological) medium), characterized by an axion term $\theta/2\pi$, such magnetically-derived vortex texture, in accordance with the Witten effect [15,16] should also carry an electronic charge that is a multiple of $\mu_0^e$ (For two fermions it should be in total of $2e$). This means that this "internal magneto-electron construction" should rather be regarded as a kind of dyon-like excitation formed by the interaction of an electric and magnetic charge located inside an electron (or a local topological medium with dimensions limited by the Compton wavelength). The assumption that inside a TI the corresponding dyon-like excitations with the implementation of the Witten effect can actually form, was expressed in [17, 18, 10-12]. In an axion medium ($\theta \neq 0$), it is the presence of an axion field that induces an electric charge attached to a magnetic charge (monopole), which is the basis for the formation of a dyon [15-18,46]. In the language of solid state physics, such a coupling could be interpreted as a coupling between two initially massless Dirac fermions of opposite chirality with internal magnetic excitation, which is an analogue of an elementary magnetic charge [17,12], that leads to the formation of the energy gap in the dispersion dependence.

The analysis carried out in Refs [10-12,46] showed that the presence of electric charge on the surface of TI should induce inside TI (as a topological medium) a magnetic texture similar to a magnetic monopole, creating the corresponding dyon-like coupling between the electric and the induced magnetic charge. This magnetic texture is similar to that of a magnetic vortex on the TI surface, formed by the electron density circulating around the core of the magnetic vortex. In Ref [12], the case of a spherical TI was considered, when the induced magnetic monopole was located inside the center of the topological sphere, and the electron density was distributed around the monopole over the surface of the sphere, with the formation of a dyon-like coupling...
between the electric charge and the induced magnetic charge. According to the estimates, such an interaction will lead to lifting the degeneracy and opening the gap in the energy spectrum of TI with a size that increases with decreasing radius of the topological sphere [12]. Such a consideration already correlates more directly with our assumption about a possible complex internal dyon-like structure of electron located inside a limited topological medium.

The idea that a magnetic monopole is associated with an electron charge and forms a magnetic texture in a topological medium with a dyon-like bond in the region of localization of an electron charge, which moves with electron under applied electric field, was expressed and analyzed in [14]. It was suggested that inside a topological medium a magnetic monopole-like texture with radial magnetic polarization \( M = aE \) can be created around each electron charge. As a result, each electron in a topological medium, in addition to charge \( e \), bears also a magnetic charge \( q \), which can significantly affect the magnetoelectric response in the system, leading to a relationship between the electric and magnetic fields \( E = aH \) and \( M = aE \) (at \( qH = eE \)). This conclusion on the relationship between a magnetic monopole and an electron in a topological medium, to some extent correlates with our assumption. However, we assume that electron itself can be characterized by an internal virtual magnetic charge localized inside electron (as topological region limited by the Compton wavelength), and itself rather has a complex dyon-like structure. It is such an internal structure of electron that can simultaneously provide the wave nature of the electron with the presence of the rest energy in the dispersion dependence.

At the same time, if we formally consider the circular motion of the electron density with the effective radius \( r_e \) using a mechanistic representation of the Bohr quantization principle \( 2\pi r_e = \lambda_n \to 2\pi \frac{c}{\omega_o} \) (at \( n = 1 \)) for the energy corresponding to the doubled electron rest energy \( 2m_o c^2 = \hbar \omega_o \), then we can get just the ratio characteristic of the Compton wavelength \( \frac{\lambda_c}{2} = r_e = \frac{c}{\omega_o} = \frac{\hbar}{2m_o c} \). This means that the concept of the electron rest energy \( (m_o c^2) \) can indeed correlate with the concept of magnetic-electron vortex with a diameter corresponding to the Compton wavelength. On the other hand, if to draw analogies with magnetic TI, then the size of the Dirac gap in a magnetic TI can be determined as \( \Delta = 2\mu_B B \frac{\hbar}{\lambda_c} \) where \( \mu_B \) is the Bohr magneton \( (\mu_B = \frac{e\hbar}{2mc}) \), \( B \) is the effective magnetic field, \( S \) is the electron spin (see, for example, [17, 39]). Based on the ratio \( \omega_o = \frac{eB}{mc} \), the expression for the effective magnetic field can be formally represented as \( B = \frac{mc \omega_o}{e} = \frac{2m c^2}{e\lambda_c^2} \) (provided that the effective radius of the circular motion of the electron density is determined by the Compton length). As a result, we obtain the following expression \( \Delta = 2 \frac{2mc^2}{e\lambda_c^2} \frac{eB}{2mc} \frac{h}{\lambda_c} = 2\frac{\mu_B h c}{\lambda_c} = 2m_o c^2 \), confirming the possibility of formal analogy between the energy gap open in magnetic TI and the electron rest energy. In this case, the electron rest energy can also be represented in the form \( \frac{\Delta}{2} = m_o c^2 = eB \frac{2mc}{2\mu_B} \). It shows that the electron rest energy can indeed be as a quantity determined by the magnitudes of the electron charge and Bohr's magneton (electrical and magnetic characteristics). Moreover, if we proceed from Dirac's postulate about the relationship between electric and magnetic charges \( eq = \frac{1}{2} \hbar c \), (see, for example [15-18]), then the value of the electron rest energy (as well as the mass of the electron) can also be related with the magnitude of the elementary magnetic charge: \( m_e c^2 = \left( \frac{c}{2} \right)^2 \frac{q}{\mu_B} \).
It should be noted here that the above representation should only be viewed as a mechanistic scaled analogy. Within the framework of quantum mechanics, the Compton wavelength exactly corresponds to the scale of the uncertainty relation. The position of an individual particle can be determined only up to the Compton length \[ \Delta x \geq \frac{1}{2} \frac{h}{mc} \]. This means that the assumed magnetic charge localized inside a topological sphere with a diameter corresponding to the Compton length must be considered as virtual. However, it manifests itself in the formation of the energy gap in the dispersion dependence of electron and determines the rest energy and mass of electron. In relation to it, the concept of the Compton length, as a region where a virtual magnetic charge can be localized, becomes fundamentally important. (The Compton length determines the distance a virtual particle can move from the point of its birth without manifesting itself explicitly [47]).

![Figure 3. Schematic representation of electron as a local topological medium with virtual internal magnetic texture limited by the sizes associated with the Compton wavelength, both in spatial and temporal coordinates, with division into temporal subspaces \((t \rightarrow +\infty)\) and \((t \rightarrow -\infty)\). Inset - The interaction of two initially massless Dirac fermions with this internal magnetic texture leads to the opening of the energy gap corresponding to the doubled electron rest energy with formation of positive and negative energies and masses located in these two temporal subspaces.](image)

In this case, the formation of an energy gap corresponding to the electron rest energy (as a consequence of the interaction of two initially massless fermions with an "internal" magnetic charge) is accompanied by the appearance of negative masses with opposite curvature in dispersion dependences and negative energies (see Figure-1). This becomes possible only if the temporal space is divided into \((t)\) and \((-t)\) subspaces characterized by the topological charges of the opposite sign (similar as it was...
analyzed for the coordinate space in [48]). In this case, the states with negative masses and energies should also be characterized by the opposite temporal directions (-t), see Figure 3. Despite this, these states are also involved in the formation of the electron rest energy for positive temporal direction, which can be considered as a half of the total energy gap between the states with positive and negative energies.

In this case, the virtual magnetic charge (monopole), located conventionally at the point t = 0 (see Figure-3), is divided into two parts acting in the subspaces (+t) and (-t), which can be conventionally considered as a combination vortex and anti-vortex, localized in subspaces (t) and (-t). As a result, Dirac fermions of opposite chirality, interacting with such a virtual magnetic excitation, mix and form the energy gap equal to the doubled electron rest energy. The size of the energy gap (and the corresponding electron rest energy) is determined by the magnitudes of the electron and magnetic charges \(m_0c^2 = (\frac{e^2}{\mu_B})\) and the dimensions of this topological structure both in temporal \((\Delta t_c = \hbar/2m_0c^2)\) and space \((\Delta r_c = \hbar/2m_0c)\) coordinates, which are related to the corresponding uncertainty relations. At larger distances and time intervals, electron can already be viewed as a particle that does not have an internal structure but is characterized by both the wave and particle properties.

Thus, the presented consideration shows a possibility of interconnection of the solutions for positive energies and masses in a subspace with a positive direction of time and the solutions with negative energies and masses in a subspace with a negative direction of time. This allows us to make a very important conclusion that availability of the rest energy (and mass) for an electron can be determined by the interaction of states of matter with positive and negative energies and masses in subspaces with negative and positive time directions. In this case, it is the violation of symmetry in the temporal space (described in this work by introducing an internal virtual magnetic charge) that determines the acquisition of mass by electron, the value of which can be represented through the corresponding electrical and magnetic parameters.

In the end we would like to note that the presented consideration does not contradict the modern concepts in the framework of the complex representation of the K-G-F field and the Higgs theory (see, for instance, [36,37]), which assume a possibility of interaction of particles and antiparticles with quantum numbers of opposite signs. This work aims to try to present possible "finger" considerations of what type of parameters and virtual processes can determine the magnitude of the electron rest energy, and ideas about the possible virtual internal structure of an electron, which, we hope, can stimulate a more detailed development of the theory in the future.

CONCLUSION

Based on the analysis of the size of the energy gap open in the TSS dispersion at the DP for magnetic TI (MnBi\(_2\)Te\(_4\)), an analogy was drawn between the size of this gap, opened by magnetic interaction, and the rest energy of electron (taking into account the replacement of the speed of light by the speed electrons at the Fermi level in magnetic TI). Simultaneously, the correlation in the interrelation was used between the energy gap and the size of magnetic vortex excitations in magnetic TI, on the one hand, and the electron rest energy with the Compton wavelength, on the other hand. This made it possible to make an assumption about the complex structure of electron, as a topological medium with the dimensions limited by the Compton wavelength and the virtual magnetic charge located inside it.
Using this analogy, it was suggested that the formation of the electron rest energy may be due to the interaction of initially massless Dirac fermions of opposite chirality with this localized virtual elementary magnetic charge. In this case, an electron itself can be considered as a complex dyon-like structure that connects the electron density with the virtual magnetic charge localized inside electron (as topological region limited by the sizes associated with the Compton wavelength). As a result, the electron rest energy is determined by the magnitudes of the electronic and magnetic charges \((m_o c^2 = \frac{e}{2} \mu_B = \frac{e^2}{2} \frac{a}{\mu_B}\) and the dimensions of the formed topological structure both in temporal \((\Delta t_c = \hbar/2m_o c^2)\) and space \((\Delta r_c = \hbar/2m_o c)\) coordinates, which are related to the corresponding uncertainty relations. It is such structure of electron, as a topological object with an internal dyon-like bonding that can simultaneously determine the wave nature of the electron with the presence of rest energy (and mass) in the dispersion dependence.

Such an interpretation includes the interconnection of the solutions for positive energies and masses in a subspace with a positive direction of time, as well as the solutions with negative energies and masses in a subspace with a negative direction of time. It allows us to make important conclusion that the acquisition of mass by electron or availability of the electron rest energy can be determined by the interaction of states with positive and negative energies and masses in subspaces with negative and positive temporal directions.

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